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SOME PROBLEMS FOR TRANSVERSAL ISOTROPIC SPACE WITH PERIODIC ANTI-CRACK PACKAGES

In this paper, the stress state of a transversely isotropic space with periodic systems (packets) of plane circular anti-cracks, the centers of which are located on the anisotropy axis, and the planes are perpendicular to it, is investigated for the first time. It is assumed that the space is under a constant biaxial compressive stress applied at infinity. Each periodic system (packet) is determined by a representative layer whose planes are perpendicular to the anisotropy axis, containing a finite system of anti-cracks of different sizes. Such a system forms a certain configuration. Any odd number of anti-cracks of arbitrary size can be included in a specific configuration, but with certain restrictions: the anti-cracks are symmetrical relative to the middle plane of the layer, their sizes satisfy a certain convergence condition. The given restrictions provide practically uniform conditions with respect to tangential stresses and normal displacements on the boundaries of the representative layer (the order of values of these quantities is in the range $10^{-10} \div 10^{-14}$), which can be considered as infinity conditions. All problems were solved by the generalized Fourier method, which allowed them to be reduced to infinite systems of linear algebraic equations with Fredholm operators. The results of the study were also based on an extensive computer experiment, within the framework of which stress distributions were calculated not only in periodic problems, but also in non-periodic problems formed by several representative layers. Practical verification of the convergence of the reduction method showed high efficiency of the generalized Fourier method. Thus, doubling the reduction parameter from 10 to 20 led to stabilization of 8 – 14 significant digits in the obtained results. Comparison of stress intensity factors for different configurations showed that for anti-cracks of the same size they depend little on a specific configuration. A qualitative conclusion that follows from the calculation results is that normal stresses on the surface of a smaller anti-crack outside its boundary in a packet increase with an increase in the size of larger neighboring anti-cracks.

Key words: transverse-isotropic space, periodic systems of anti-cracks, compressed spheroidal coordinates, generalized Fourier method, representative layer, Fredholm operator, stress intensity factor.

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ДЕЯКІ ЗАДАЧІ ДЛЯ ТРАНСВЕРСАЛЬНО-ІЗОТРОПНОГО ПРОСТОРУ З ПЕРІОДИЧНИМИ ПАКЕТАМИ АНТИТРІЩИН

У роботі вперше досліджено напружений стан трансверсально-ізотропного простору з розташованими в ньому періодичними системами (пакетами) плоских кругових антитріщин, центри яких знаходяться на осі анізотропії, а їх площини перпендикулярні до неї. Вважається, що простір знаходиться під сталим двовісним стискаючим напруженням, прикладеним на нескінченності. Кожна періодична система (пакет) антитріщин визначається представницьким шаром, площини якого перпендикулярні до осі анізотропії, і який містить скінченну кількість антитріщин різного розміру. Такий пакет формує певну конфігурацію. У конкретну конфігурацію можна включити будь-яку непарну кількість антитріщин довільного розміру, але з певними обмеженнями: антитріщини симетричні відносно середньої площини шару, їх розміри задовольняють певній умові збіжності. Наведені обмеження забезпечують практично однорідні умови відносно дотичних напружень і нормальних переміщень на межах представницького шару (порядок значень цих величин знаходиться в діапазоні $10^{-10} \div 10^{-14}$), які можна розглядати як умови на нескінченності. Усі задачі розв'язувалися узагальненим методом Фур'є, що дозволило звести їх до нескінченних систем лінійних алгебраїчних рівнянь з фредгольмовими операторами. Результати дослідження також ґрунтувалися на широкому комп'ютерному експерименті, в рамках якого розраховувалися розподіли напружень не лише в періодичних задачах, а й у неперіодичних задачах, утворених кількома представницькими шарами. Практична перевірка збіжності методу редукції показала високу ефективність узагальненого методу Фур'є. Так, подвоєння параметра редукції з 10 до 20 призвело до стабілізації 8 – 14 значущих цифр в отриманих результатах. Порівняння коефіцієнтів інтенсивності напружень для різних конфігурацій показує, що для антитріщин однакового розміру вони мало залежать від конкретної конфігурації. Якісний висновок, який випливає з результатів розрахунку, полягає в тому, що нормальні напруження на поверхні меншої за розміром антитріщини поза її межею в пакеті зростають із збільшенням розміру більших сусідніх антитріщин.

Ключові слова: трансверсально-ізотропний простір, періодична система антитріщин, стиснуті сфероїдальні координати, узагальнений метод Фур'є, представницький шар, фредгольмів оператор, коефіцієнт інтенсивності напружень.

Introduction. One of the key tasks of modern science is aimed at solving the problem of creating optimal materials in which certain physical and mechanical properties would be combined with a relatively simple technology and an acceptable cost of their production. An important class of such materials are composites, which have found wide use in various branches of technology. For them, it is the selection of the characteristics of the structural components that determines the features of the resulting material. For this reason, an important place in the mechanics of composite materials is occupied by mathematical modeling, which is aimed at accumulating facts about the behavior of the stress-strain state of bodies having a certain structure near various stress concentrators: inclusions, cavities, cracks, anti-cracks, etc. When assessing the strength characteristics of such materials, it is necessary to have an idea not only about the distribution of stresses near individual inhomogeneities, but also about the mutual influence of various concentrators on the overall stress state. One of the possibilities to take into account the structure of the composite is to model it by a periodic system of inhomogeneities. This work is devoted precisely to periodic systems of anti-cracks in transversely isotropic space.

Review of the results of recent research. Numerical and analytical methods in problems of crack and anti-crack theory are actually the same or similar, therefore the review of studies considers works devoted to both types of inhomogeneities, especially since, for obvious reasons, cracks are given more attention. Research in this direction was developed by several authors. In [1], an axisymmetric problem of a circular subsurface radial shear crack in a semi-infinite composite material with initial stresses was investigated using singular integral equations. The problem is reduced to a system of Fredholm integral equations of the second kind. A representation of the stress intensity coefficients around the crack tip depending on the initial stresses was obtained. For two types of composite materials (layered composites with isotropic layers and composites stochastically reinforced with short ellipsoidal fibers), the stress intensity coefficients were calculated and their dependence on the initial stresses, physical and mechanical characteristics of the composites, and geometric parameters of the problem were investigated. In the article [2], the axisymmetric problem of the failure of a prestressed composite material with a periodic system of parallel coaxial cracks of normal separation is investigated. Using representations of general solutions of linearized equilibrium equations through harmonic potential functions and the apparatus of Hankel integral transformations, the problem is reduced to a system of paired integral equations, and then to a solvable Fredholm integral equation of the second kind. The work [3] is devoted to the asymptotic analysis of stress in an isotropic material near the boundary of circular cracks, anti-cracks, thin inclusions under different conditions on its surface. In the local coordinate system associated with the edge of inhomogeneity, asymptotic solutions are constructed in the form of expansions by eigenfunctions that depend on the angular coordinates and power series by the radial variable. Many approximate formulas for stress intensity factors have been obtained, but there are no numerical results. In the study [4], an exact solution to the problem of a circular interfacial crack in a piecewise homogeneous transversely isotropic space under the action of arbitrary loads applied to the crack boundary was constructed by the method of integral transformations. Formulas for the stress intensity coefficients at the crack boundary and the values of these coefficients for some combinations of transversely isotropic materials were obtained. In the article [5], the problems of the theory of cracks located near the surfaces of volumetric and thin-walled bodies under thermal and force static and dynamic loads are investigated using the method of thin inclusions proposed by the authors. The influence of body surfaces or the interface of its materials on static and dynamic coefficients of stress intensity around defects is described. The dissertation [6] uses the technique of separation of variables, integral transformations and methods of solving double and triple integral equations to solve a number of mixed problems of crack theory. The article [7] investigates the singularity of stresses and displacements near a crack within the limits of the simplified Gurtin-Murdoch linear model of surface elasticity. The technique of Mellin and Wiener-Hopf integral transformations is used. In some works, the Green's function apparatus is used to study the stress state around cracks. Thus, in the article [8], Green's functions for an infinite three-dimensional elastic body containing a circular crack were derived through integrals of elementary functions. A solid is considered to be either isotropic or transversely isotropic with a crack parallel to the plane of isotropy. In [9], integral equations of the problem of the interaction of parallel circular cracks under arbitrary loading in a transversely isotropic elastic space were derived using the Green's function. The theorem on the average value of the integral was used to highlight the singularities associated with the crack tips. After that, the equations lose their singularity and can be solved numerically. Numerical results are given only for stress intensity coefficients. The potential theory method was applied in [10] to solve the problem of thermoelasticity for an isotropic space with an anti-crack under the influence of a temperature field. The singular integral equations for an anti-crack of arbitrary shape are derived in terms of unknown thermal shear stress jumps. A similar approach was used in [11] to obtain an analytical solution to a three-dimensional transversely isotropic thermoelastic problem in which a uniform heat flow acts on a space with a circular anti-crack. The problems of constructing numerical and analytical solutions in problems with cracks arbitrarily oriented in relation to the axis of anisotropy led to the creation of approximate models for the stress state of such bodies. Thus, in work [12] an approximate analytical model of the inclusion of an arbitrarily oriented circular crack in the effective elastic compliance of a transversely isotropic material is considered. The application of the hypothesis that the change in the elastic potential due to an arbitrarily oriented circular crack in a transversely isotropic material can be approximated by the change calculated for a certain isotropic medium is investigated. The article [13] focuses on the calculation of the general elastic properties of a transversely isotropic material containing several randomly oriented circular cracks. A new methodology is proposed for estimating the contribution of one arbitrarily oriented crack in an infinite transversely isotropic medium to the overall modulus of elasticity. The paper uses the Mori–Tanaka–Benveniste scheme, which coincides with the interaction-free approximation for the case of crack-like inhomogeneities. The Fourier method in problems of the theory of elasticity for transversely isotropic bodies with one canonical inclusion or cavity was considered in works [14 – 16]. The development and application of the generalized Fourier method for transversely isotropic doubly connected bodies, the centers of whose boundary surfaces coincide, were considered in works [17, 18]. Two parallel cracks in the transversally isotropic space were considered in the article [19], where the basicity of the constructed solutions was also shown.

The given review of literary sources devoted to research in the theory of cracks and anti-cracks shows the importance and relevance of further study of the stress state of interacting cracks, as well as the development of a mathematical apparatus for its implementation. It also demonstrates the lack of research on periodic packages of cracks and anti-cracks.

This paper considers problems for periodic systems of plane parallel circular anti-cracks in transversally isotropic space. Each system is defined by a specific set of anti-cracks located in a representative layer of space, the planes of which are parallel to the crack planes and the isotropy plane. It is assumed that such a layer periodically extends over the entire space. The choice of the number of anti-cracks, the distances between them and their sizes in the representative layer can be arbitrary if the conditions of symmetry of the anti-cracks relative to the median plane of the representative layer and convergence of the method are met. Next, three options are considered: one anti-crack in the representative layer, three anti-cracks with two different sizes, five anti-cracks with three sizes.

General formulation of the problem. Consider an elastic transversely isotropic space with an infinite system of uniaxial parallel plane circular cracks $\{\Gamma_l\}_{l=-\infty}^{\infty}$. Let's mark the centers of the anti-cracks $\{O_l\}_{l=-\infty}^{\infty}$, the distance between them h ($h>0$), anti-crack radii a_l. We will assume that the axis of anisotropy of the transversely isotropic space passes through the centers of the cracks. Let's fix in space the Cartesian coordinate system (x, y, z) and the cylindrical system (ρ, φ, z) associated with it so that the point O_0 is the common origin, and the axis Oz has a directional vector $\overrightarrow{O_0O_1}$. The elastic steels of the space material are denoted by constants $\{c_{ij}\}_{i,j=1}^{3}$. We will consider these constants to be positive. For basic transversally isotropic materials, this condition is fulfilled. Consider the problem of determining the stress state of the space indicated above in the case when a constant compressive biaxial stress is applied at infinity. The problem boils down to the solution of the boundary value problem for the system of equations of equilibrium of a transversely isotropic body, which in the axisymmetric formulation can be written as follows:

$$\left[c_{11}\left(\Delta_2-\frac{1}{\rho^2}\right)+c_{44}\frac{\partial^2}{\partial z^2}\right]V_\rho+(c_{13}+c_{44})\frac{\partial^2 V_z}{\partial\rho\partial z}=0, \tag{1}$$

$$\left[c_{44}\Delta_2+c_{33}\frac{\partial^2}{\partial z^2}\right]V_z+(c_{13}+c_{44})\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V_\rho}{\partial z}\right)=0, \tag{2}$$

$$(x, y, z)\in\Omega\equiv\mathbb{R}^3\setminus\bigcup_{l=-\infty}^{\infty}\Gamma_l .$$

The boundary conditions on the anti-crack surfaces and at infinity have the form:

$$\vec{V}(x, y, z)_{|(x,y,z)\in\Gamma_l}=0,\ l=-\infty,\infty,\ \sigma_\rho^\infty=-\sigma,\ \tau_{\rho\varphi}^\infty=0,\ \tau_{\rho z}^\infty=0. \tag{3}$$

Above (V_ρ, V_z), $(\sigma_z, \tau_{\rho z}, \tau_{\varphi z})$ – are the components of the axisymmetric displacement vector and stress tensor in cylindrical coordinates, $\Delta_2\equiv\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}$ – an axisymmetric variant of the two-dimensional Laplace operator in polar coordinates.

To construct partial solutions of the system of equations (1), (2), which correspond to the geometry of the domain Ω, with each point O_l we will connect the equally directed with the coordinate system $Oxyz$ local Cartesian coordinate system (x_l, y_l, z_l), and also two Cartesian $\{(x_{ls}, y_{ls}, z_{ls})\}_{s=1}^{2}$ and two oblate spheroidal $\{(\tilde{\xi}_{ls}, \tilde{\eta}_{ls}, \varphi)\}_{s=1}^{2}$ coordinate systems such that:

$$x=x_j=x_{js}=a_j\mathbf{ch}\tilde{\xi}_{js}\sin\tilde{\eta}_{js}\cos\varphi,\quad y=y_j=y_{js}=a_j\mathbf{ch}\tilde{\xi}_{js}\sin\tilde{\eta}_{js}\sin\varphi,\quad \frac{z_j}{\sqrt{\nu_s}}=z_{js}=a_j\mathbf{sh}\tilde{\xi}_{js}\cos\tilde{\eta}_{js}.$$

Here a_l is the parameter of the spheroidal system, which coincides with the radius of the corresponding anti-crack, $\tilde{\xi}_{ls}\in[0,\infty)$, $\tilde{\eta}_{ls}\in[0,\pi]$, $\varphi\in[0,2\pi]$, the equation of the $l-$th crack surface is $\tilde{\xi}_{ls}=0$. The parameter ν_s is the root of the equation

$$c_{11}c_{44}\nu^2-(c_{11}c_{33}-2c_{13}c_{44}-c_{13}^2)\nu+c_{33}c_{44}=0. \tag{4}$$

In the following, we will consider the case when the roots of equation (4) are real, positive and different It follows from the relations between the coordinates that equalities $\tilde{\eta}_{l1}=\tilde{\eta}_{l2}=\tilde{\eta}_l$ are fulfilled on the surface of the $l-$ th anti-crack.

In [17], the sets of linearly independent partial solutions of the general system of equilibrium equations in displacements in a oblate spheroidal coordinate system were constructed in the form of basis vector functions. In the case of an axisymmetric stress state, we obtain from those solutions:

$$\vec{V}_{s,n}^{\pm(6)}(\tilde{\xi}_{js}, \tilde{\eta}_{js})=\frac{-ia_j}{2n+1}\vec{\nabla}_s[u_{n-1}^{\pm(6)}(\tilde{\xi}_{js}, \tilde{\eta}_{js})-u_{n+1}^{\pm(6)}(\tilde{\xi}_{js}, \tilde{\eta}_{js})],\quad n=0,1,...,\ s=1,2, \tag{5}$$

where

$$u_n^{\pm(6)}(\xi,\eta)=\begin{Bmatrix}Q_n(i\mathbf{sh}\tilde{\xi})\\ P_n(i\mathbf{sh}\tilde{\xi})\end{Bmatrix}P_n(\cos\tilde{\eta});\ \vec{\nabla}_s=\vec{e}_\rho\frac{\partial}{\partial\rho}+k_s\vec{e}_z\frac{\partial}{\partial z},\ k_s=\frac{c_{11}\nu_s-c_{44}}{c_{13}+c_{44}},\ s=1,2,$$

$P_n(x), Q_n(x)$ – Legendre functions of the first and second kind; $\{\vec{e}_\rho, \vec{e}_z\}$ – unit base vectors of the cylindrical coordinate system. The displacements (5) in coordinates have the following form:

$$\vec{V}_{s,n}^{\pm(6)}(\tilde{\xi}_{js}, \tilde{\eta}_{js})=u_n^{\pm(6)1}(\tilde{\xi}_{js}, \tilde{\eta}_{js})\vec{e}_\rho-\frac{k_s}{\sqrt{\nu_s}}u_n^{\pm(6)}(\tilde{\xi}_{js}, \tilde{\eta}_{js})\vec{e}_z,\ s=1,2, \tag{6}$$

where

$$u_n^{\pm(6)1}(\tilde{\xi},\tilde{\eta})=\begin{Bmatrix} Q_n^1(i\mathbf{sh}\tilde{\xi}) \\ P_n^1(i\mathbf{sh}\tilde{\xi}) \end{Bmatrix} P_n^{-1}(\cos\tilde{\eta}).$$

Building a solution to the problem in a general setting. We will look for a solution to problem (1) – (3) with conditions at $\rho\to\infty$ in the form

$$\vec{V}(x,y,z)=\vec{V}_0(x,y,z)+\sum_{s=1}^{2}\sum_{l=-\infty}^{\infty}\sum_{n=0}^{\infty}A_{s,n}^{(l)}\vec{V}_{s,n}^{+(6)}(\tilde{\xi}_{ls},\tilde{\eta}_{ls}), \tag{7}$$

where $A_{s,n}^{(l)}$ are unknown coefficients that must be found in the process of solving the problem, $\vec{V}_0(x,y,z)$ – displacement that meets the conditions at infinity.

We find the vector function $\vec{V}_0$ as a solution to the system of equations (1), (2) in the form $\vec{V}_0=B\rho\vec{e}_\rho$. Then, at $B=-\sigma/(c_{11}+c_{12})$, the displacement $\vec{V}_0$ sets a uniform stress state in space, which corresponds to the boundary conditions at infinity (3)

We will use the result proved in [19].

Theorem 1. *Under condition* $\sqrt{\nu_s}a_{ls}+\sqrt{\nu_s}a_{ms}\mathbf{ch}\tilde{\xi}_{ms}<|m-l|h$, $l,m=-\infty\div\infty$, $l\neq m$, *the following addition theorem holds*:

$$V_{s,n}^{+(6)}(\tilde{\xi}_{ls},\tilde{\eta}_{ls})=\sum_{k=0}^{\infty}V_{s,k}^{-(6)}(\tilde{\xi}_{ms},\tilde{\eta}_{ms})\omega_{n,k}^{l,m}\sum_{j=n}^{\infty}g_{n,m-l}^{(64)j}(a_{ls})f_{j,m-l}^{(46)k}(a_{ms}), \tag{8}$$

where

$$g_{n,m-l}^{(64)j}(a_{ls})=\frac{\sqrt{\pi}\varepsilon_{j,n}}{\mathbf{\Gamma}(j/2-n/2+1)\mathbf{\Gamma}(j/2+n/2+3/2)}\left(-\frac{i}{2}\frac{\sqrt{\nu_s}a_{ls}}{|m-l|h}\right)^{j+1},$$

$$f_{j,m-l}^{(46)k}(a_{ms})=\sum_{p=k}^{\infty}\frac{\sqrt{\pi}(k+1/2)\varepsilon_{p,k}(p+j)!}{\mathbf{\Gamma}(p/2-k/2+1)\mathbf{\Gamma}(p/2+k/2+3/2)}\left(\frac{i}{2}\frac{\sqrt{\nu_s}a_{ms}}{|m-l|h}\right)^{p},$$

$$\varepsilon_{n,k}=\begin{cases}1, & n-k=2p,\ p\in\mathbb{Z};\\ 0, & n-k=2p+1,\ p\in\mathbb{Z},\end{cases}\quad \omega_{n,k}^{l,m}=[\mathbf{sign}(m-l)]^{n+k},$$

$\mathbf{\Gamma}(x)$ – *Euler's gamma function*.

Let's transform the displacement vector (7) using formula (8) to each individual coordinate system. As a result, we have:

$$\vec{V}(x_l,y_l,z_l)=Ba_l\mathbf{ch}\tilde{\xi}_{l,s}\sin\tilde{\eta}_{l,s}\vec{e}_\rho+$$
$$+\sum_{s=1}^{2}\sum_{n=0}^{\infty}A_{s,n}^{(l)}\vec{V}_{s,n}^{+(6)}(\tilde{\xi}_{ls},\tilde{\eta}_{ls})+\sum_{s=1}^{2}\sum_{n=0}^{\infty}\vec{V}_{s,n}^{-(6)}(\tilde{\xi}_{ls},\tilde{\eta}_{ls})\sum_{m\neq l}\sum_{k=0}^{\infty}\omega_{n,k}^{m,l}A_{s,k}^{(m)}\sum_{j=k}^{\infty}g_{k,m-l}^{(64)j}(a_{ms})f_{j,m-l}^{(46)n}(a_{ls}). \tag{9}$$

Passing to the coordinate form of displacements in (9) and satisfying the boundary conditions (3) on the surface Γ_l, we obtain a resolving system

$$\sum_{s=1}^{2}\left[\tilde{A}_{s,n}^{(l)}+\sum_{m\neq l}\sum_{k=0}^{\infty}t_{0,n,k}^{s,m,l}\tilde{A}_{s,k}^{(m)}\right]=-\frac{4}{\pi}\delta_{n,1},\ n=0\div\infty,\ l=-\infty\div\infty, \tag{10}$$

$$\sum_{s=1}^{2}\frac{k_s}{\sqrt{\nu_s}}\left[\tilde{A}_{s,n}^{(l)}+\sum_{m\neq l}\sum_{k=0}^{\infty}t_{1,n,k}^{s,m,l}\tilde{A}_{s,k}^{(m)}\right]=0,\ n=0\div\infty,\ l=-\infty\div\infty, \tag{11}$$

where

$$t_{0,n,k}^{s,m,l}=i^n\frac{2}{\pi}\frac{a_m}{a_l}\sin(\pi n/2)h_{m,l,n,k}^{(66)s},\ t_{1,n,k}^{s,m,l}=i^{n+1}\frac{2}{\pi}\frac{a_m}{a_l}\cos(\pi n/2)h_{m,l,n,k}^{(66)s};$$

$$A_{s,n}^{(l)}=Ba_l\tilde{A}_{s,n}^{(l)},\ h_{m,l,n,k}^{(66)s}=\omega_{n,k}^{m,l}\sum_{j=k}^{\infty}g_{k,m-l}^{(64)j}(a_{ms})f_{j,m-l}^{(46)n}(a_{ls}),$$

$\delta_{n,k}$ – Kronecker delta symbol.

Note that at $n=0$, system (10) – (11) is satisfied at $\tilde{A}_{s,n}^{(l)}=0$ $(s=1,2)$, which is a necessary condition for the regularity of solution (7).

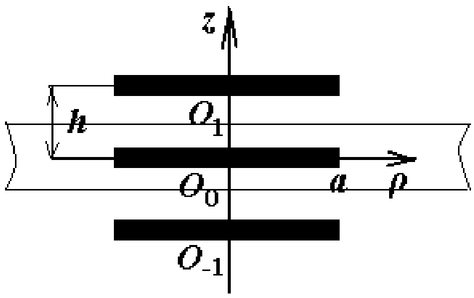

Fig. 1 – Representative layer. Configuration 1.

Solving problems with a periodic system of anti-cracks. In order to formulate a periodic problem, it is necessary to clarify what is meant by such a problem. Note that not for every system of anti-cracks that is periodically repeated can be correctly formulated a periodic problem. In [20], the idea of a representative layer in a thermoelastic problem for a periodic system of spherical inclusions was proposed, on the boundaries of which conditions were chosen that allowed periodic continuation to infinity. Usually, these are some homogeneous conditions that can be realized at infinity. However, if in the article [20] the periodic problem was replaced by an equivalent problem for a representative layer, then in this work, periodic problems are solved directly, and the conditions on the boundaries of the representative layer are verified numerically (their fulfillment follows from a certain symmetry of the problems under consideration).

Configuration 1. Consider a system of identical anti-cracks (configuration 1). A representative layer for such a system is shown in Fig. 1. Since the problem is periodic, the unknown coefficients $\tilde{A}_{s,n}^{(l)}$ should not depend on the index l. Then the system (10), (11) can be rewritten in the following form:

$$\sum_{s=1}^{2}\left[\tilde{A}_{s,n}+\sum_{k=1}^{\infty}\tilde{A}_{s,k}\sum_{m\neq 0}t_{0,n,k}^{s,m,0}\right]=-\frac{4}{\pi}\delta_{n,1},\ n=1\div\infty, \tag{12}$$

$$\sum_{s=1}^{2}\frac{k_s}{\sqrt{\nu_s}}\left[\tilde{A}_{s,n}+\sum_{k=1}^{\infty}\tilde{A}_{s,k}\sum_{m\neq 0}t_{1,n,k}^{s,m,0}\right]=0,\ n=1\div\infty. \tag{13}$$

Theorem 2. *When the condition* $a_{ls}+a_{ms}<h/\sqrt{\nu_s},\ l\neq m,\ s=1,2$ *is met the operator of system (12), (13) is a Fredholm operator in Hilbert space* $l_2\times l_2$.

Proof. To prove the theorem, it is enough to show the absolute convergence of the series

$$\sum_{n,k=1}^{\infty}\sum_{m\neq l}t_{r,n,k}^{s,m,l}\quad r=0,1;\ s=1,2;\ l=-\infty\div\infty.$$

Let's mark

$$Z_{s,m,l}=\frac{1}{2}\frac{\sqrt{\nu_s}\,a_{ms}}{|m-l|\,h}.$$

Consider a series

$$\sum_{m\neq l}\sum_{n,k=1}^{\infty}\sum_{j=k}^{\infty}|\,g_{k,m-l}^{(64)j}(a_{ms})\,||\,f_{j,m-l}^{(46)n}(a_{ls})\,|.$$

It can be transformed by replacing the summation orders and subscripts with a series of form

$$\sum_{m\neq l}\sum_{j,p=1}^{\infty}(p+j)!Z_{l,m}^{j+1}Z_{m,l}^{p}\sum_{s=1}^{[j/2]}\frac{\sqrt{\pi}}{s!\Gamma(j-s+3/2)}\sum_{r=1}^{[p/2]}\frac{\sqrt{\pi}(p-2r+1/2)}{r!\Gamma(p-r+3/2)}.$$

We will use estimates

$$\sum_{s=1}^{[j/2]}\frac{1}{s!\Gamma(j-s+3/2)}\leq\sum_{s=1}^{[j/2]}\frac{1}{s!\Gamma(j-s+1)}\leq\sum_{s=0}^{j}\frac{1}{s!(j-s)!}=\frac{2^j}{j!},$$

$$\sum_{r=1}^{[p/2]}\frac{(p-2r+1/2)}{r!\Gamma(p-r+3/2)}\leq\sum_{r=1}^{p-1}\frac{1}{r!(p-1-r)!}\leq\frac{2^{p-1}}{(p-1)!}.$$

As a result, the original series is majorized by the series

$$\sum_{m\neq l}\sum_{j,p=1}^{\infty}(p+j)!\frac{2^j}{j!}Z_{l,m}^{j+1}\frac{2^{p-1}}{(p-1)!}Z_{m,l}^{p}=\frac{1}{4}\sum_{m\neq l}\sum_{j,p=1}^{\infty}\frac{(p+j)!}{j!(p-1)!}\left(\frac{\sqrt{\nu_s}\,a_{ms}}{|m-l|\,h}\right)^{j+1}\left(\frac{\sqrt{\nu_s}\,a_{ls}}{|m-l|\,h}\right)^{p}=$$

$$=\frac{1}{4}\sum_{j,p=1}^{\infty}\frac{(p+j)!}{j!(p-1)!}\left(\frac{\sqrt{\nu_s}\,a_{ms}}{h}\right)^{j+1}\left(\frac{\sqrt{\nu_s}\,a_{ls}}{h}\right)^{p}\sum_{m\neq l}\frac{1}{|m-l|^{j+p+1}}<\frac{1}{4}\sum_{j=0,p=1}^{\infty}\frac{(p+j)!}{j!(p-1)!}\left(\frac{\sqrt{\nu_s}\,a_{ms}}{h}\right)^{j+1}\left(\frac{\sqrt{\nu_s}\,a_{ls}}{h}\right)^{p}2\sum_{t=1}^{\infty}\frac{1}{t^2}=$$

$$=\frac{\pi^2}{12}\sum_{j=0,p=1}^{\infty}\frac{(p+j)!}{j!(p-1)!}\left(\frac{\sqrt{\nu_s}\,a_{ms}}{h}\right)^{j+1}\left(\frac{\sqrt{\nu_s}\,a_{ls}}{h}\right)^{p}.$$

The last series converges for $a_{ls}+a_{ms}<h/\sqrt{\nu_s}$.

The solution of the system was used to determine the distribution of stress in the region of its maximum concentration – in the plane of the anti-crack outside its boundary. The following formulas were obtained for the stress σ_z and *stress intensity factor (SIF)* K_{1z}:

$$\sigma_z(\rho,0)(c_{11}+c_{12})/(\sigma c_{44}) = -2\frac{c_{13}}{c_{44}} - \sum_{s=1}^{2}(k_s+1)\sum_{n=1}^{\infty}\tilde{A}_{s,n}\left\{\sum_{m\neq 0}\frac{1}{(\operatorname{sh}^2\tilde{\xi}_{sm}+\cos^2\tilde{\eta}_{sm})}\times\right.$$

$$\left.\times\lim_{x\to\infty}\left[\operatorname{sh}\tilde{\xi}_{sm}\sin\tilde{\eta}_{sm}Q_n(i\operatorname{sh}\tilde{\xi}_{sm})P_n^1(\cos\tilde{\eta}_{sm}) - \operatorname{ch}\tilde{\xi}_{sm}\cos\tilde{\eta}_{sm}Q_n^1(i\operatorname{sh}\tilde{\xi}_{sm})P_n(\cos\tilde{\eta}_{sm})\right] + \frac{1}{\operatorname{sh}\tilde{\xi}_{s0}}Q_n(i\operatorname{sh}\tilde{\xi}_{s0})P_n^1(0)\right\},$$

$$K_{1z}(c_{11}+c_{12})/(c_{44}\sigma\sqrt{a}) = -\sqrt{\pi}\sum_{s=1}^{2}(k_s+1)\sum_{n=0}^{\infty}\tilde{A}_{s,2n+1}.$$

Numerical results for all considered configurations were obtained for the material of the space, which is sandstone with elastic constants $c_{11} = 5.8576\cdot 10^{10}$ Pa, $c_{12} = 2.5019\cdot 10^{10}$ Pa, $c_{13} = 2.0793\cdot 10^{10}$ Pa, $c_{33} = 6.1105\cdot 10^{10}$ Pa, $c_{44} = 1.6584\cdot 10^{10}$ Pa. For it, the roots of equation (4) are equal $\nu_1 = 0.52$, $\nu_2 = 2.01$.

In Fig. 3 shows the stress distribution in the plane of the anti-crack outside its boundary depending on the relative size of the anti-crack in the representative layer. With a decrease in the relative size of the anti-crack, the magnitude of the stress decreases, and in the vicinity of the anti-crack boundary, a change in the sign of the stress is observed (the point of sign change is not shown on the graph due to its proximity to the boundary $\rho = a$). In this figure and in all others, it is indicated by σ^* the stress distribution for one anti-crack in the entire space. It is given for comparison, since the problem for one anti-crack has an exact solution in closed form

$$\sigma_z(\rho,0)(c_{11}+c_{12})/(\sigma c_{44}) = -2\frac{c_{13}}{c_{44}} - \frac{4}{\pi}\frac{k_1(k_2+1)\sqrt{\nu_2}-k_2(k_1+1)\sqrt{\nu_1}}{k_2\sqrt{\nu_1}-k_1\sqrt{\nu_2}}\left[\frac{1}{\sqrt{(\rho/a)^2-1}} - \arcsin(a/\rho)\right],$$

$$K_{1z}(c_{11}+c_{12})/(c_{44}\sigma) = -\frac{4\sqrt{a}}{\sqrt{\pi}}\frac{k_1(k_2+1)\sqrt{\nu_2}-k_2(k_1+1)\sqrt{\nu_1}}{k_2\sqrt{\nu_1}-k_1\sqrt{\nu_2}}.$$

Table 1 – Dependence of SIF on crack sizes. Configuration 1

a/h	0.0	0.1	0.2	0.3
$K_{1z}(c_{11}+c_{12})/(c_{44}\sigma)$	1.1529	1.1534	1.564	1.1609

The smallest stress value corresponds to one anti-crack in space. This is natural, since one anti-crack is the limiting case of configuration 1, when a is fixed and $h\to\infty$. The stress intensity factor has a similar nature of change. Table 1 shows the dependence of stress intensity coefficients (SIN) on the parameter a/h. Here, the value $a/h = 0.0$ corresponds to the limiting case $h\to\infty$, that is, one crack in space.

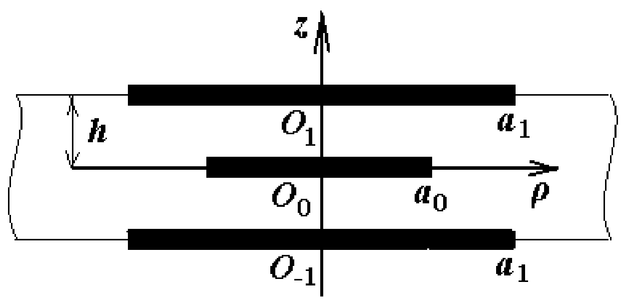

Fig. 2 – Representative layer. Configuration 2.

Configuration 2. Consider another configuration of cracks in the representative layer (Fig. 2). We denote it as $a_0 - a_1 - a_0 - a_1$.

The solution of problem (1) – (3) for configuration 2 has the form (7), in which

$$A_{s,n}^{(2l+j)} = A_{s,n}^{(j)},\ a_{2l+j} = a_j,\ j = 0,1;\ l = -\infty\div\infty.$$

The resolving system here is written as follows:

$$\sum_{s=1}^{2}\left[\tilde{A}_{s,n}^{(l)} + \sum_{j=0}^{1}\sum_{k=1}^{\infty}\tilde{A}_{s,k}^{(j)}\sum_{m:\,2m+j\neq l}t_{0,n,k}^{s,2m+j,l}\right] = -\frac{4}{\pi}\delta_{n,1},\ n = 1\div\infty,\ l = 0,1, \tag{14}$$

$$\sum_{s=1}^{2}\frac{k_s}{\sqrt{\nu_s}}\left[\tilde{A}_{s,n}^{(l)} + \sum_{j=0}^{1}\sum_{k=1}^{\infty}\tilde{A}_{s,k}^{(j)}\sum_{m:\,2m+j\neq l}t_{1,n,k}^{s,2m+j,l}\right] = 0,\ n = 1\div\infty,\ l = 0,1. \tag{15}$$

The proof of the Fredholm property of the system operator for (14), (15) is similar to the proof of Theorem 2, so it is not given here. The results of the calculations are shown in Fig. 4 and Table 2.

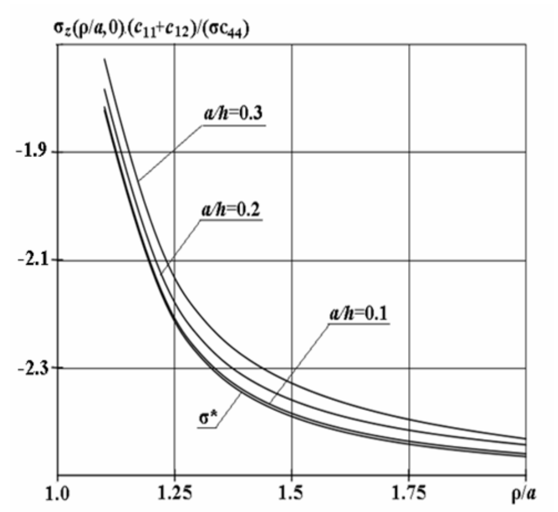

Fig. 3 – Stress distribution $\sigma_z(\rho,0)(c_{11}+c_{12})/(\sigma c_{44})$ depending on relative crack size. Configuration 1.

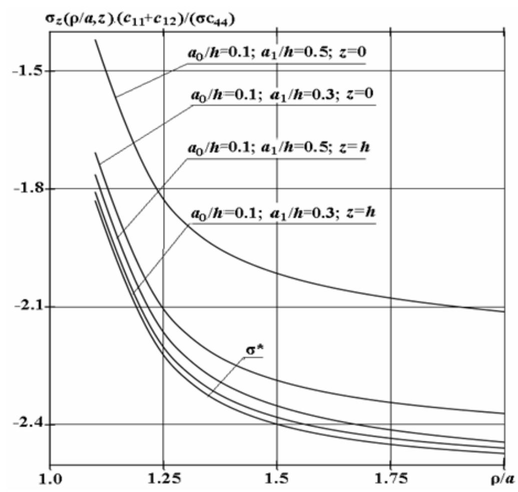

Fig. 4 – Stress distribution $\sigma_z(\rho,0)(c_{11}+c_{12})/(\sigma c_{44})$ depending on relative crack size. Configuration 2.

The proof of the Fredholm property of the system operator for (14), (15) is similar to the proof of Theorem 2, so it is not given here. The results of the calculations are shown in Fig. 4 and Table 2.

Anti-cracks of different sizes in the representative layer already influence each other when calculating the intensity factors and the distribution of stresses near the boundaries of anti-cracks. An increase in the size of a larger anti-crack with a fixed size of a smaller one leads to an increase in the SIF at the boundary of a smaller anti-crack. Conversely, an increase in the size of a smaller anti-crack with a fixed size of a larger one leads to a decrease in the SIF at the boundary of a larger anti-crack. The same patterns are observed in the distributions of normal stresses in the planes of anti-cracks outside their boundaries.

Table 2 – Dependence of SIF on crack sizes. Configuration 2

$(a_0/h, a_1/h)$	$(0.1, 0.3)$	$(0.2, 0.4)$	$(0.1, 0.5)$
$K_{1z}(c_{11}+c_{12})/(c_{44}\sigma),\ z=0$	1.1630	1.1692	1.1785
$K_{1z}(c_{11}+c_{12})/(c_{44}\sigma),\ z=h$	1.1549	1.1577	1.1587

Configuration 3. Now consider configurations with three different cracks. The representative layer for the first of them is shown in Fig. 5. Let's denote it as $a_0-a_1-a_2-a_1-a_0$.

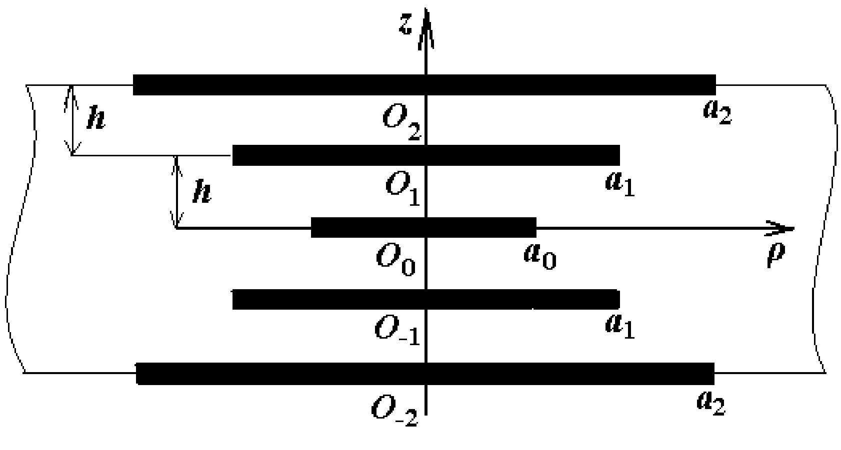

Fig. 5 – Representative layer. Configuration 3.

The solution of problem (1) – (3) for configuration 4 has the form (7), in which

$$A_{s,n}^{(4l+j)}=A_{s,n}^{(j)},\ a_{4l+j}=a_j,\ j=0\div 3,\ l=-\infty\div\infty,\ a_1=a_{-1},\ a_2=a_{-2}.$$

The resolving system for this problem is as follows:

$$\sum_{s=1}^{2}\left[\tilde{A}_{s,n}^{(l)}+\sum_{j=0}^{3}\sum_{k=1}^{\infty}\tilde{A}_{s,k}^{(j)}\sum_{m:\,4m+j\neq l}t_{0,n,k}^{s,4m+j,l}\right]=-\frac{4}{\pi}\delta_{n,1},\quad n=1\div\infty,\ l=0\div3, \tag{16}$$

$$\sum_{s=1}^{2}\frac{k_s}{\sqrt{\nu_s}}\left[\tilde{A}_{s,n}^{(l)}+\sum_{j=0}^{3}\sum_{k=1}^{\infty}\tilde{A}_{s,k}^{(j)}\sum_{m:\,4m+j\neq l}t_{1,n,k}^{s,4m+j,l}\right]=0,\quad n=1\div\infty,\ l=0\div3. \tag{17}$$

The results of the calculations are shown in Fig. 6, Fig. 7 and Table 3.

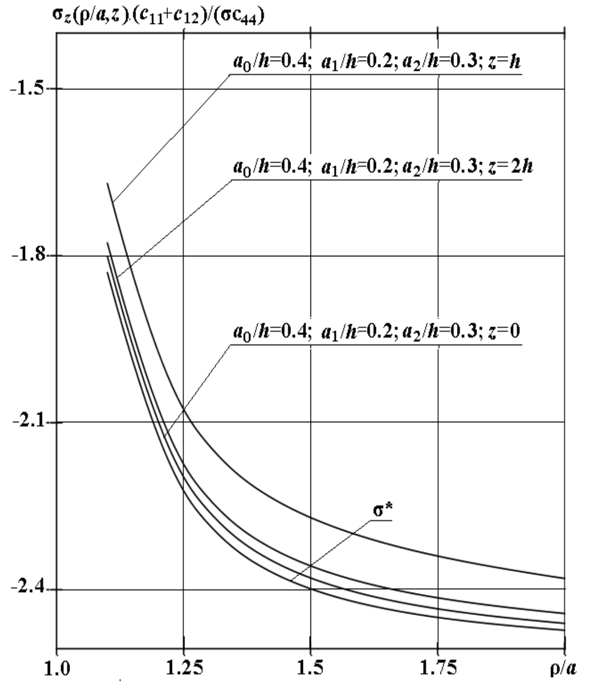

Fig. 6 – Stress distribution $\sigma_z(\rho,0)(c_{11}+c_{12})/(\sigma c_{44})$ depending on relative crack size. Configuration 3.

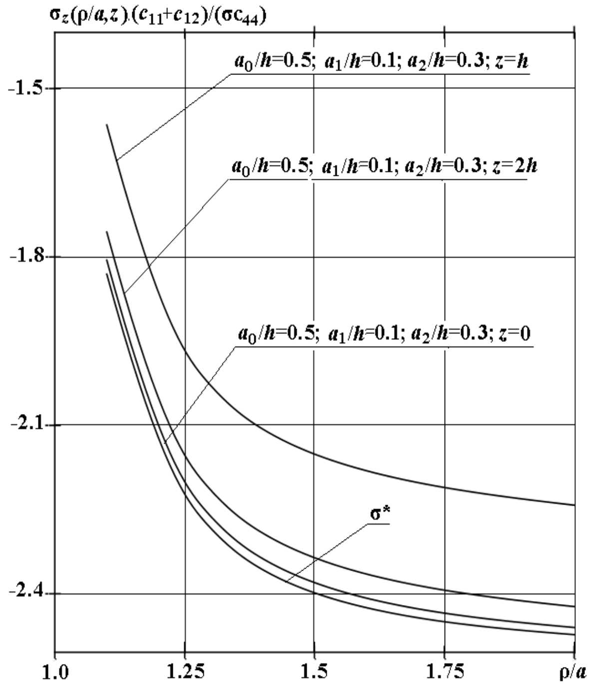

Fig. 7 – Stress distribution $\sigma_z(\rho,0)(c_{11}+c_{12})/(\sigma c_{44})$ depending on relative crack size. Configuration 2.

For each package of anti-cracks of configuration 3, the highest SIF is observed at the boundary of the smallest anti-crack, and with an increase in the size of neighboring anti-cracks, this SIF increases. Similar patterns were obtained in the distribution of stresses $\sigma_z(\rho,0)(c_{11}+c_{12})/(\sigma c_{44})$ in the planes of smaller anti-cracks outside their boundaries in a package with large anti-cracks.

Table 3 – Dependence of SIF on crack sizes. Configuration 3

$(a_0/h, a_1/h, a_2/h)$	$(0.2, 0.1, 0.3)$	$(0.4, 0.2, 0.3)$	$(0.5, 0.1, 0.3)$
$K_{1z}(c_{11}+c_{12})/(c_{44}\sigma),\ z=0$	1.1549	1.1561	1.1551
$K_{1z}(c_{11}+c_{12})/(c_{44}\sigma),\ z=h$	1.1597	1.1656	1.1707
$K_{1z}(c_{11}+c_{12})/(c_{44}\sigma),\ z=2h$	1.1539	1.1586	1.1594

Conclusions. In this paper, the stress-strain state of a transversely isotropic space with periodic packets of flat circular anti-cracks with their centers located on the anisotropy axis and their planes perpendicular to it is investigated for the first time. It is assumed that the space is under the action of a constant biaxial compressive stress applied at infinity. The periodic system (packet) of anti-cracks is determined by a representative layer with planes perpendicular to the anisotropy axis and containing anti-cracks of various sizes. Such a package forms a certain configuration. Any odd number of anti-cracks of arbitrary size can be included in a specific configuration, subject to the following restrictions: the anticracks are symmetrical with respect to the median plane of the layer, their sizes satisfy the convergence conditions of the method. The established restrictions provide practically uniform conditions for tangential stresses and normal displacements on the boundary of the representative layer (the order of these quantities is in the range $10^{-10} \div 10^{-14}$), which can be considered as conditions at infinity. All problems were solved using the generalized Fourier method, which allowed them to be reduced to infinite systems of linear algebraic equations with Fredholm operators. Practical verification of the reduction method efficiency showed high efficiency of the generalized Fourier method. Thus, increasing the reduction parameter from 10 to 20 led to stabilization of 8 – 14 significant digits in the obtained results. Comparison of stress intensity factors for different configurations shows that for anti-cracks of the same size, their values depend little on a specific configuration. One of the parallel lines of research of similar problems is connected with periodic packets of cracks in transversely isotropic space. Recently the authors of this article have carried out such researches and their results are in print. Another promising direction of research is the class of non-axisymmetric problems with periodic systems of cracks and anti-cracks.

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