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O. B. AKHIEZER, H. O. HOLOTAYSTROVA, Y. P. GOMOZOV, V. I. MATC, A. I. ROGOVYI

STRATEGIC MANAGEMENT OF THE PORTFOLIO OF FINANCIAL ASSET

Until now, three approaches to asset portfolio management have been used. The first approach is the classic one, based on the "Efficient Market Hypothesis" (EMH). The second and more modern approach is related to the "Fractal Market Hypothesis" (FMH). Modern economic practice is characterized by the presence of structurally unstable markets, included as nodes in the network of the world economy, which functions in real time. The structure of the available financial instruments is heterogeneous and non-Markovian processes arise in them. The third approach is the formation of a dynamic strategy of investment management of the asset portfolio. Due to the complex structure of the modern global financial market, the heterogeneous structure of available financial instruments and traders using different approaches and time horizons, forecasts, as a rule, require a large number of observations, work poorly at the edges of bifurcations, and do not have a computer model that could build forecasts in real time. In these structures, slow diffusion-type processes with a memory phenomenon occur, i.e., non-Markovian processes. Therefore, the formation of a dynamic strategy using modern methods of mathematical and computer modeling is very promising.

Real financial data is expressed in rational numbers. Computational models were developed on the basis of classical substance analysis. Therefore, p – adic analysis methods are increasingly used in financial mathematical modeling. These methods are used, in particular, in the construction of neural networks, cellular automata, and percolation models. The paper seems to have taken the first step towards building a "synthetic" model of dynamic asset portfolio management. The model has the form of a differential equation in fractional derivatives obtained using the so-called interbasin kinetics method. A general form of the energy market model is also offered, as one of the specific, especially nowadays, markets.

Key words: portfolio of financial assets, fractal environment, memory phenomenon, p – adic analysis, differential equation in fractional derivatives, interbasin kinetics.

О. Б. АХІЄЗЄР, Г. О. ГОЛОТАЙСТРОВА, Є. П. ГОМОЗОВ, В. І.МАЦ, А. І. РОГОВИЙ

СТРАТЕГІЧНЕ УПРАВЛІННЯ ПОРТФЕЛЕМ ФІНАНСОВИХ АКТИВІВ

До цього часу використовувалися три підходи до управління портфелем активів. Перший підхід – класичний, заснований на «Гіпотезі ефективного ринку» (ГЕР). Другий і більш сучасний підхід пов'язаний з «гіпотезою фрактального ринку» (ГФР). Сучасна економічна практика характеризується наявністю структурно нестабільних ринків, включених як вузли в мережу світового господарства, що функціонує в режимі реального часу. Структура наявних фінансових інструментів неоднорідна і в них виникають немарківські процеси. Третій підхід – формування динамічної стратегії інвестиційного управління портфелем активів. Через складну структуру сучасного світового фінансового ринку, неоднорідну структуру доступних фінансових інструментів і трейдерів, що використовують різні підходи та часові горизонти, прогнози, як правило, вимагають великої кількості спостережень, погано працюють на краях біфуркацій і не мають комп'ютерної моделі, яка могла б будувати прогнози в реальному часі. У цих структурах відбуваються повільні процеси дифузійного типу з явищем пам'яті, тобто немарківські процеси. Тому дуже перспективним є формування динамічної стратегії з використанням сучасних методів математичного та комп'ютерного моделювання. Реальні фінансові дані виражаються в раціональних числах. Обчислювальні моделі розроблено на основі класичного субстанційного аналізу. Тому у фінансово-математичному моделюванні дедалі ширше використовуються методи p – адичного аналізу. Ці методи використовуються, зокрема, при побудові нейронних мереж, клітинних автоматів, перколяційних моделей. Стаття, здається, зробила перший крок до побудови «синтетичної» моделі динамічного управління портфелем активів. Модель має вигляд диференціального рівняння у дробових похідних, отриманого за допомогою так званого методу міжбасейнової кінетики. Запропоновано також загальну форму моделі енергетичного ринку, як одного із специфічних, особливо в наш час, ринків.

Ключові слова: портфель фінансових активів, фрактальне середовище, феномен пам'яті, p – адичний аналіз, диференціальне рівняння у дробових похідних, міжбасейнова кінетика.

Е. Б. АХИЕЗЕР, Г. А. ГОЛОТАЙСТРОВА, Е. П. ГОМОЗОВ, В. И. МАЦ, А. И. РОГОВОЙ

СТРАТЕГИЧЕСКОЕ УПРАВЛЕНИЕ ПОРТФЕЛЕМ ФИНАНСОВЫХ АКТИВОВ

До сих пор использовались три подхода к управлению портфелем активов. Первый подход – классический, основанный на «Гипотезе эффективного рынка» (ГЭР). Второй и более современный подход связан с «гипотезой фрактального рынка» (ГФР). Современная экономическая практика характеризуется наличием структурно неустойчивых рынков, включенных как узлы в сеть мировой экономики, которая функционирует в режиме реального времени. Структура имеющихся финансовых инструментов неоднородна и в них возникают немарковские процессы. Третий подход заключается в формировании динамической стратегии управления инвестициями портфеля активов. Из-за сложной структуры современного мирового финансового рынка, неоднородной структуры доступных финансовых инструментов и трейдеров, использующих разные подходы и временные горизонты, прогнозы, как правило, требуют большого количества наблюдений, плохо работают на гранях бифуркаций и не имеют компьютерной модели, которая могла бы строить прогнозы в режиме реального времени. В этих структурах протекают медленные процессы диффузионного типа с явлением памяти, то есть немарковские процессы. Поэтому формирование динамической стратегии с использованием современных методов математического и компьютерного моделирования весьма перспективно. Реальные финансовые данные выражаются рациональными числами. Вычислительные модели были разработаны на основе классического анализа вещества. Поэтому методы p – адического анализа находят все более широкое применение в финансово-математическом моделировании. Эти методы используются, в частности, при построении нейронных сетей, клеточных автоматов и моделей перколяции. В статье, похоже, сделан первый шаг к построению «синтетической» модели динамического управления портфелем активов. Модель имеет вид дифференциального уравнения в дробных производных, полученного с помощью так называемого метода межбассейновой кинетики. Предлагается также общая форма модели энергетического рынка, как одного из специфических, особенно в настоящее время, рынков.

Ключевые слова: портфель финансовых активов, фрактальная среда, феномен памяти, p – адический анализ, дифференциальное уравнение в дробных производных, межбассейновая кинетика.

Introduction. The *stock market* has always been an important part *of economic processes*. That is, it was responsible for the relations that arise between market subjects within the framework *of investment processes*. From this point of view, the stock market was seen as local, and such that mathematical models could use probability theory and mathematical statistics. Investment decisions regarding optimal portfolio management did not need to be made in real time. Moreover, optimal portfolios were created by stock traders within the framework of the so-called "investment style", i.e. from securities of the same type.

However, after the advent of the Internet, the stock market became global. Figuratively speaking, now the global stock market is, in some sense, a guidepost of the global world economy. Modern economic practice is primarily characterized by the presence of structurally unstable markets, included as nodes in the network of the global economy, which functions in real time. Thus, investment decisions must also be made in real time. It follows from the theory of dynamic systems that the network of modern financial markets, functioning in real time, should include structurally unstable systems and stable areas of chaos. As numerous studies have shown, now market returns do not follow a normal distribution, the market can be very volatile.

Thus, the task of creating new mathematical models of strategic management of a portfolio of various financial assets (financial instruments) is very urgent.

Analysis of research and publications. *Securities* determine the relationship between the person who placed (issued) them and the owner, and also provide for the possibility of transferring the rights arising from these documents to other persons.

Investing in securities involves an assessment of the financial result − an increase or decrease in equity as a result of investment, taking into account possible risks. It is well known, that, in the classic sense, the investment process is based on the following procedure: choosing an investment style, analyzing the securities market, forming a portfolio, reviewing the portfolio, evaluating the portfolio's effectiveness. Thus, modeling the value of financial assets allows the investor to choose exactly which securities and in what quantity will be included in his portfolio, as well as to develop a successful strategy for managing the securities portfolio.

The most important principle of investing is that the value of an asset changes over time. The time for which an investor places the investment capital is called an investment horizon. At the same time, although investment capital has a well-defined value at the initial moment in time, its future value at the initial moment is unknown.

It is assumed that investors make decisions in "risk-return" coordinates. Investors who work within the above approaches are usually called "rational".

Securities, as one of the types of exchange goods, are divided into two groups: primary securities and derived securities or derivatives. The basis of primary securities are assets to which the securities themselves do not belong. *Derivative securities (derivatives)* are contracts that determine the rights and obligations of the parties regarding the underlying asset (for example, the primary security), which is the basis of this financial instrument, in the future and lead to a positive or negative financial result for each party.

Investors who try to profit by changing asset prices are called "speculators". "Speculators" with a short investment horizon are called "scalpers".

Investors who contribute to the inflation of "soap bubbles" are usually called "noise". For example, the modern cryptocurrency market is a market of "noise" investors; they are also largely responsible for the financial crashes of the stock markets.

Portfolio analysis. Most investors choose to invest in more than one financial instrument, that is, form a certain combination of them, thus obtaining an investment portfolio.

At the stage of formation, the portfolio is a set of financial assets owned by the investor. The assets were selected based on the analysis of the securities market. In modern practice, a portfolio can include instruments of the same type or different assets, for example, securities, futures contracts, real estate.

However, for further calculations, the portfolio is understood as a vector, the coordinates of which are the shares of the full investment in the portfolio of already selected assets:

$$\vec{x} = \left(x_1, x_2, \ldots, x_n\right), \quad \sum_{i=1}^{n} x_i = 1 .$$

If the investor uses only his own funds when investing, then such a portfolio is called standard, and the class of admissible portfolios forms a standard $(n-1)$ simplex. If the investor also uses loan funds when investing, then the class of admissible portfolios forms an affine $(n-1)$ – dimensional hyperplane.

Classical models of all portfolio theories require the fulfillment of certain hypotheses for the asset market.

Based on the above approaches, the so-called *portfolio theory of Markowitz* emerged, which assumes that the market has a probabilistic nature, the value of assets and their returns are random variables, and the risk of changes in returns is a standard deviation. With the appearance of the classic work of Markowitz in 1954, the variance was initially used as a measure of risk, making the problem of portfolio optimization a problem of quadratic programming, and not a mathematical one, as in the case of using standard deviation as a measure of risk. Within the framework of Markowitz's portfolio theory, three options for creating an optimal portfolio are used: portfolio risk is limited from above and profitability is maximized; portfolio profitability is limited from below and risk is minimized; the so-called "utility function" is constructed and the optimization problem is solved for it.

Based on Markowitz's classic model, *the Efficient Market Model (EMH)* was created, which has a large number of assumptions and limitations, including the so-called "typical investor" behavior model. But there is a very large number of mathematical models that are based, precisely, on the EMH hypothesis.

The Fractal Market Model (FMH) was created as an alternative to the EMH and is currently the most advanced. This hypothesis emphasizes the influence of information and investment horizons on the behavior of investors. It is assumed that people do not recognize and react to trends until those trends are well established and investors make decisions based on accumulated but previously ignored information.

This behavior is fundamentally different from the behavior of a rational investor, who, according to the EMH, immediately uses new information to make investment decisions. The main tools of FMH are fractal geometry and chaotic systems theory. This is due to the fact that fractals allow describing unstable systems, and the need to apply the theory of chaotic systems arises when analyzing financial data over a long period of time.

There have been attempts to create new or modified hypotheses of the behavior of stock markets - a synergistic model, a multi-agent model, a modification of the EMH based on the so-called "probability game" theory.

Bayesian methods of risk assessment and profitability forecasting of financial instrument portfolios are described in [1]. The simulation results are presented in the form of probability distribution histograms, which allows for a detailed analysis of the "risk-profit" ratio.

Bayesian methodology is actually broader than a set of ways to handle conditional probabilities in directed graphs. It also includes models with symmetric connections, models of dynamic processes, and a wide class of models with hidden variables that allow solving probabilistic problems of classification, pattern recognition, and prediction.

For many years, *neural networks* have been used in financial trading [2]. This has become widespread due to the fact that the use of neural networks allows finding patterns in heterogeneous data, in conditions of information noise and uncertainty. Moreover, modern economic practice is characterized by the presence of structurally unstable markets included as nodes in the real-time network of the world economy.

In [3] $p-$ *adic cellular neural networks* are considered, which have a large number of hidden layers and arise as boundaries of large hierarchical neural networks.

In [4], based on the *model of cellular automata*, the processes taking place in the stock market obey the rules of transactions adopted in China. The market under consideration is artificial and the forecasting results are based on both technical methods and investor behavior and interaction.

The application of *percolation theory to financial market bubbles and crashes* is being researched at the Department of Entrepreneurial Risk at the Zurich University of Technology under the direction of *Professor Didier Sornette*.

Since all raw data in the stock markets are expressed in rational numbers, and the price charts do not depend on the choice of number systems, it is not surprising that quite a large number of publications are devoted to cluster analysis on a $p-$ adic basis and its use in the selection of portfolios of securities as, for example, in [5, 6].

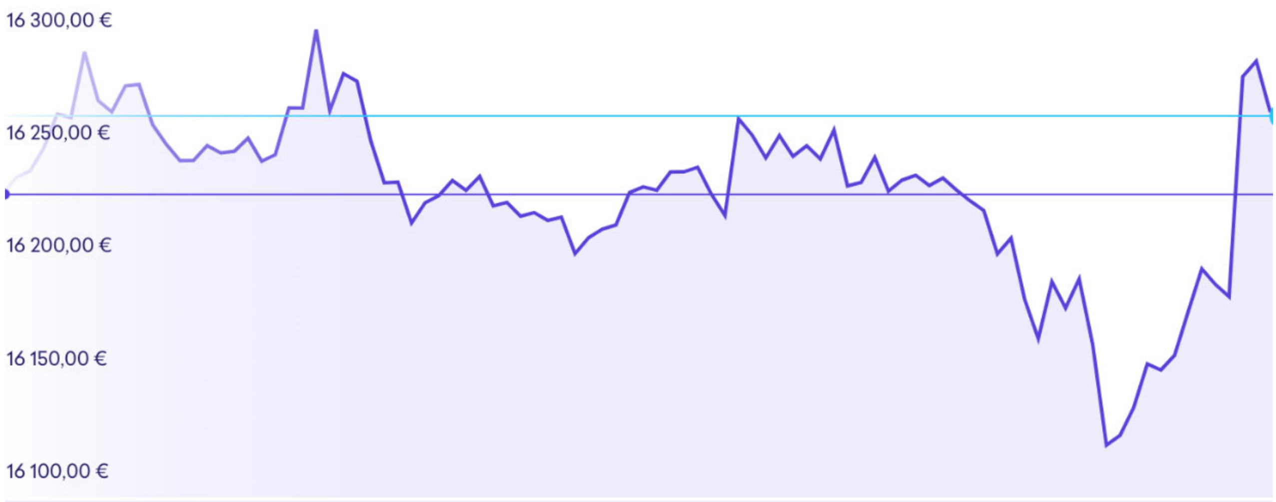

Fig. 1 – Bitcoin BTC price change during the day.

Theoretical and practical issues of frequency $p-$ adic probability theory are considered, for example, in [7, 8]. The result can also be used to analyze the profitability of an investment portfolio.

Finally, in [9] the *Shannon entropy* was considered in fractional terms, which allows us to use the results of this work to build a model.

Another approach that allows monitoring and analyzing complex landscapes with a large number of maxima and minima is *the interbasin kinetics method* [10]. They are usually used in chemistry and physics, but the graph of changes in the price of assets on the stock market is also quite complex (see, for example, Figure 1). Therefore, the methods of interbasin kinetics, in a certain interpretation, can be used to build a model for managing a portfolio of financial assets.

The portfolio of derivatives was studied in [11]. To hedge the risks of a sharp change in futures prices, a portfolio of futures as the underlying asset and hedge options of these futures is compiled. Derivatives have an "average" lifetime, and for this market, the risk of a collapse of rates is also less significant compared to the indeterminacy of rate fluctuations.

In this work, for the management of such a portfolio, the possibility of using the percolation method algorithms to calculate the fractal dimension of the relevant time series of derivatives rates is added.

For portfolio management, a modification of *the well-known Hutchinson-Wright equations* was used in [11]:

$$D^{\gamma}_{\alpha+}\left(F\right)=\lambda F\left[1-D^{\alpha}\left(F\right)\right]-\mu FD\ ;$$

$$D^{\gamma}_{\beta+}\left(O\right)=-\varepsilon O+\nu OG\ ,$$

where λ, μ, ν, ε are positive parameters; $\nu>\varepsilon$, μ are bifurcation parameter; $F\left(N,t\right)$ denotes the offer price on the futures market, which is expressed as a $p-$adic number; $O\left(N,t\right)$ stands for the offer price in the options market, which is expressed as a $p-$adic number, N the NASDAQ index value; t denotes time; α is the minimum offer price for sale on the futures market, which is expressed as a $p-$adic number; β is the minimum bid price on the options market, which is expressed as a $p-$adic number; $D^{\gamma}_{\alpha+}, D^{\gamma}_{\beta+}$ are *the Riemann-Liouville fractional derivatives*; D^{α} is the Riesz fractional derivative.

This model has a single value of the equilibrium price and, in general, is not finite-dimensional.

The purpose of the study. It should be noted that all known mathematical formalizations of the functioning of the investment asset market use certain models of diffusion-type processes in a stationary environment. However, such processes are "memoryless". By the way, this property is one of the main postulates of technical analysis. On the other hand, data on exchange rates of past periods are used to forecast exchange rates. This contradiction is implicitly tried to be eliminated by different methods in different approaches to forecasting. In addition, as a rule, a specific market for specific assets is considered (rather than the currently existing global one). Currently, one should consider the global market of heterogeneous assets with an almost infinite set of agents with different types of behavior and different investment horizons. Note that the effects associated with the topological structures of this market have not yet been studied. The observed behavior of the modern market does not fit into the standard hypotheses about the independence of random variables in diffusion-type equations. Perhaps, therefore, it would be appropriate to define risks as Shannon's fractional information entropy.

Mathematical model. We present a mathematical model of dynamic management of a portfolio of financial assets.

As part of dynamic portfolio management, we will change its structure in real time to improve the portfolio's investment properties. Based on some approaches of the Elliott wave theory, one can assume the presence of spatial and temporal non-locality of asset return series. This presence of long-range action leads to the appearance of the phenomenon of memory, that is, to non-Markov dynamics. Therefore, in our opinion, the following approaches should be combined:

1. A model describing portfolio profitability. It should be described by a diffusion-type equation in fractional derivatives with real time and $p-$adic incomes;

2. Any series of asset returns contains a large number of local highs and lows. Therefore, it makes sense to use the methods of "interbasin kinetics" in the appropriate financial interpretation;

3. Since the corresponding equation above takes into account the $p-$adic probability of the $p-$adic distance between the reference lines of the yield plot, it is possible to interpret the solution of the equation as a utility function.

Taking into account all these considerations, we will write a dynamic model for optimizing portfolio profitability according to well-known equations from medical dynamics as a "binding" of the weights of the vector of portfolio assets (the $p-$adic equation of diffusion with a drain created by *the fractional Vladimirov operator*):

$$\left[\frac{\partial}{\partial t}+D^{\alpha}_{x}+\Omega\left(\left|x\right|_{p}\right)\right]I\left(x,t\right)=0\ .$$

Here $I(x,t)$ is the expected portfolio return; x is the vector of weights of portfolio assets; α denotes the Hurst exponent of the set of asset price graphs of the portfolio; $\mu(x)$ stands for Haar's measure; $\Omega\left(|x|_p\right)$ is the characteristic function on the standard simplex in $\mathbb{Q}_p$; D_x^α is the fractional Vladimirov operator:

$$D_x^\alpha I(x,t) = \Gamma_p^{-1}(-\alpha)\int_{\mathbb{Q}_p} \frac{I(x,t)-I(y,t)}{|x-y|_p^{1+\alpha}} d\mu(x).$$

There are quite a few methods of numerical solution of equations in fractional derivatives: for example, the method based on the definitions of Riemann-Liouville and *Caputo*, ordinary-difference approximations based on the definition in the sense of Grunwald-Letnikov.

In our opinion, the most convenient to use is the modification of the difference scheme, known as *Euler's method* [12], since this scheme is unconditionally stable under any parameters of fractional derivatives.

A special place among assets, especially at the present time, is occupied by the energy market.

We propose a nonlinear model of the most general energy market that meets the following requirements:

1. Assumes that the cost of electricity includes the cost of services and financial rights.
2. Assumes that the rate of price change depends on weighted average prices in the past.
3. Assumes that the weighted average price in the capacity market is a known function.
4. The model has the possibility of hedging the risks of sudden price changes by a portfolio of futures and options.
5. There are control parameters that can be determined empirically.

Then we get the integro-differential model:

$$\dot{q}(t) = \lambda q(t)\left[1 - \int_{-\infty}^{0} q(t+\tau)w(-\tau)d\tau\right] - \mu p(t)q(t),$$

$$p(t) = -\alpha p(t) + \beta p(t)q(t) - f\left(p(t),t\right),$$

where λ, μ, α, β are positive parameters; $\alpha > \beta$, μ are bifurcation parameter; $q(t)$ is weighted average price of the offer of the wholesale electricity market; $p(t)$ denotes weighted average price of the offer of the retail electricity market; $w(t)$ is weighted average offer price of the wholesale capacity market; $f\left(p(t),t\right)$ stands for the option premium price.

Comparing these equations with the above-written equations of the derivative portfolio management model, we can conclude that further on we need to move on to equations in fractional derivatives using fractional operators in some optimal sense for this case.

Obtained results. Mathematical models of dynamic portfolio management in the form of differential equations in fractional derivatives were obtained.

A nonlinear model of the most general energy market was also obtained. This model has a single value of the equilibrium price. For a kernel $w(t)$ of the general form, the system is not finite-dimensional. But in specific cases, the model can be reduced to a system of four ordinary differential equations. In such cases, the cycles born at the value λ_0 of the parameter λ have asymptotic orbital constancy.

Prospects for further research. In the future, it is quite possible to improve the proposed dynamic model of portfolio management by developing correct methods for selecting the factors that have the greatest impact on the forecast, as well as combining it with other methods of forecasting of rates and trend reversal points.

Classical methods of evaluating financial derivatives do not work well for energy markets. Therefore, the task of building models for evaluating "energy" futures, options, and hedge portfolios is urgent.

The problems of application and computer implementation of integro-differential models have not yet been sufficiently resolved. The study of such models is an important and relevant direction, and the development of methods and tools for their computer simulation is of importance for a wide range of applied problems.

Conclusions. This work is an introduction to the creation of software implementations of predictive dynamic models capable of working in real time.

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Відомості про авторів / Сведения об авторах / Information about authors

Ахієзер Олена Борисівна – кандидат технічних наук, професорка, завідувачка кафедри комп’ютерної математики та аналізу даних, Національний технічний університет «Харківський політехнічний інститут», м. Харків; тел.: (057) 707-63-51; e-mail: Olena.Akhiiezer@khpi.edu.ua.

Ахиезер Елена Борисовна – кандидат технических наук, профессор, заведующая кафедры компьютерной математики и анализа данных, Национальный технический университет «Харьковский политехнический институт», г. Харьков; тел.: (057) 707-63-51; e-mail: Olena.Akhiiezer@khpi.edu.ua.

Akhiiezer Olena Borisovna – Candidate of Technical Sciences, Professor, Head of the Department of Computer Mathematics and Data Analysis, National Technical University «Kharkiv Polytechnic Institute», Kharkiv; tel.: (057) 707-63-51; e-mail: Olena.Akhiiezer@khpi.edu.ua.

Голотайстрова Галина Олександрівна – старший викладач кафедри комп'ютерної математики та аналізу даних, Національний технічний університет «Харківський політехнічний інститут», м. Харків; тел.: (057) 707-63-51; e-mail: Halina.Holotaistrova@khpi.edu.ua.

Голотайстрова Галина Александровна – старший преподаватель кафедры компьютерной математики и анализа данных, Национальный технический университет «Харьковский политехнический институт», г. Харьков; тел.: (057) 707-63-51; e-mail: Halina.Holotaistrova@khpi.edu.ua.

Holotaistrova Halyna Oleksandrivna – Senior Lecturer of the Department of Computer Mathematics and Data Analysis, National Technical University «Kharkiv Polytechnic Institute», Kharkiv; tel.: (057) 707-63-51; e-mail: Halina.Holotaistrova@khpi.edu.ua.

Гомозов Євген Павлович – кандидат фізико-математичних наук, доцент кафедри комп'ютерної математики та аналізу даних, Національний технічний університет «Харківський політехнічний інститут», м. Харків; тел.: (057) 707-63-51; e-mail: Yevgen.Gomozov@khpi.edu.ua.

Гомозов Евгений Павлович – кандидат физико-математических наук, доцент кафедры компьютерной математики и анализа данных, Национальный технический университет «Харьковский политехнический институт», г. Харьков; тел.: (057) 707-63-51; e-mail: Yevgen.Gomozov@khpi.edu.ua.

Gomozov Yevgen Pavlovych – Candidate of Physical and Mathematical Sciences, Associate Professor of the Department of Computer Mathematics and Data Analysis, National Technical University «Kharkiv Polytechnic Institute», Kharkiv; tel.: (057) 707-63-51; e-mail: Yevgen.Gomozov@khpi.edu.ua.

Мац Владислав Ігорович – аспірант кафедри комп'ютерної математики та аналізу даних, Національний технічний університет «Харківський політехнічний інститут», м. Харків; тел.: (057) 707-63-51; e-mail: vladyslav.mats@cs.khpi.edu.ua.

Мац Владислав Игоревич – аспирант кафедры компьютерной математики и анализа данных, Национальный технический университет «Харьковский политехнический институт», г. Харьков; тел.: (057) 707-63-51; e-mail: vladyslav.mats@cs.khpi.edu.ua.

Mats Vladyslav Igorovych – Graduate Student of the Department of Computer Mathematics and Data Analysis, National Technical University «Kharkiv Polytechnic Institute», Kharkiv; tel.: (057) 707-63-51; e-mail: vladyslav.mats @cs.khpi.edu.ua.

Роговий Антон Іванович – кандидат технічних наук, доцент кафедри стратегічного управління, Національний технічний університет «Харківський політехнічний інститут», м. Харків; тел.: (057) 707-68-32; e-mail: anton.rogovyi@khpi.edu.ua.

Роговой Антон Иванович – кандидат технических наук, доцент кафедры стратегического управления, Национальный технический университет «Харьковский политехнический институт», г. Харьков; тел.: (057) 707-68-32; e-mail: anton.rogovyi@khpi.edu.ua.

Rogovyi Anton Ivanovych – Candidate of Technical Sciences, Associate Professor of Strategic Management Department, National Technical University «Kharkiv Polytechnic Institute», Kharkiv; tel.: (057) 707-68-32; e-mail: anton.rogovyi@khpi.edu.ua.