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STUDYING THE BEHAVIOR OF RANK (QUASIRANK) AND INFINITESIMAL CORRELATION FUNCTIONS OR CORRELATION DIFFERENCES IN LINEAR TRANSFORMATIONS OF RANDOM FUNCTIONS

The first-order linear stochastic equation $x(n+1) = ax(n) + b\xi(n)$, $x(n)|_{n=0} = x_0$ determines the simplest kind of regression signal that is widely used in applications. The case where the right part is a non-stationary sequence has not actually been investigated. In the paper the properties of the solution of this equation are studied within the framework of the correlation theory in the case when $\xi(n)$ belongs to a particular class of random non-stationary signals, in addition, the classification is carried out using the concepts of rank or quasirank of non-stationarity. The Hilbert approach to the correlation theory of random sequences utilized in the paper allows us to study the question of the asymptotic behavior of the correlation function and makes it possible to obtain a simple inhomogeneous representation of the correlation function in terms of the correlation difference.

Key words: correlation function, mathematical expectation, non-stationary random sequences and processes, rank of non-stationarity.

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ДОСЛІДЖЕННЯ ПОВЕДІНКИ РАНГУ (КВАЗИРАНГУ) ТА ІНФІНІТЕЗИМАЛЬНИХ КОРЕЛЯЦІЙНИХ ФУНКЦІЙ АБО КОРЕЛЯЦІЙНИХ РІЗНИЦЬ ПРИ ЛІНІЙНИХ ПЕРЕТВОРЕННЯХ ВИПАДКОВИХ ФУНКЦІЙ

Лінійне різницеве стохастичне рівняння першого порядку $x(n+1) = ax(n) + b\xi(n)$, $x(n)|_{n=0} = x_0$ визначає найпростіший вид регресійного сигналу, який широко використовується в застосуваннях. Випадок, коли права частина є нестационарною послідовністю, фактично не досліджувався. В статті досліджуються властивості розв'язку цього рівняння в межах кореляційної теорії у випадку, коли $\xi(n)$ належить тому чи іншому класу випадкових нестационарних сигналів, до того ж класифікація здійснюється за допомогою понять рангу або квазірангу нестационарності. Гільбертів підхід до кореляційної теорії випадкових послідовностей, використаний у статті, дозволяє досліджувати питання про асимптотичну поведінку кореляційної функції і отримати просте однозначне представлення кореляційної функції через кореляційні різниці.

Ключові слова: кореляційна функція, математичне очікування, нестационарні випадкові послідовності і процеси, ранг нестационарності.

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ИССЛЕДОВАНИЕ ПОВЕДЕНИЯ РАНГА (КВАЗИРАНГА) И ИНФИНИТЕЗИМАЛЬНЫХ КОРЕЛЯЦИОННЫХ ФУНКЦИЙ ИЛИ КОРЕЛЯЦИОННЫХ РАЗНОСТЕЙ ПРИ ЛИНЕЙНЫХ ПРЕОБРАЗОВАНИЯХ СЛУЧАЙНЫХ ФУНКЦИЙ

Линейное разностное стохастическое уравнение первого порядка $x(n+1) = ax(n) + b\xi(n)$, $x(n)|_{n=0} = x_0$ определяет простейший вид регрессионного сигнала, который широко используется в приложениях. Случай, когда правая часть представляет собой нестационарную последовательность, фактически не исследовался. В статье исследуются свойства решения этого уравнения в рамках корреляционной теории в случае, когда $\xi(n)$ принадлежит тому или иному классу случайных нестационарных сигналов, причем классификация осуществляется с помощью понятий ранга или квазіранга нестационарности. Гильбертов подход к корреляционной теории случайных последовательностей, использованный в статье, позволяет исследовать вопрос об асимптотическом поведении корреляционной функции и получить простое однозначное представление корреляционной функции через корреляционную разность.

Ключевые слова: корреляционная функция, математическое ожидание, нестационарные случайные последовательности и процессы, ранг нестационарности.

Introduction. To solve applied problems, it is often necessary to consider deterministic linear transformations of random functions and find appropriate probability characteristics (mixed moments of various order, distribution laws). For instance, a necessity might arise in finding statistical characteristics of the output of a linear filter if the input is subjected to a random influence or determining statistical characteristics of an electromagnetic field on a chaotic screen, etc.

Analysis of recent research. An important role in applications is played by the class of linear transformations that preserves the properties of stationarity (homogeneity) [1 – 4]. For *non-stationary (inhomogeneous)* random functions, the structure of linear transformation did not play any role, since there were no characteristics that would describe deviations from *stationarity (homogeneity)*. The introduction in [8, 9] of infinitesimal correlation functions and correlation differences necessitated the consideration of new problems related to the study of the behavior of these functions in the linear transformations of random processes, sequences and fields. Obviously, with an arbitrary linear transformation of a stationary (homogeneous) random function, a new random function arises, which has, generally speaking, *an infinite rank (quasirank)*. Therefore, as in the case of linear transformations of stationary random functions, we should restrict ourselves to the corresponding classes of linear transformations [5, 6].

Formulation of the problem. To simulate a wide range of processes with discrete time, stochastic linear difference equations are used with a random right-hand part representing a discrete white noise [3, 7]. The case where the

right-hand part is a sequence belonging to a class of non-stationary random sequences has not actually been investigated. Also the nature of non-stationarity has not been studied. Given the above, the relevant task is to study the properties of the solution of linear difference stochastic equation of the first order (1) within the framework of correlation theory in the case when it belongs to a class of random non-stationary signals, in addition, classification is carried out by means of rank concepts [8] or quasirank non-stationarity [9].

Mathematical model. We begin by considering the linear first-order difference stochastic equation:

$$x(n+1) = ax(n) + b\xi(n), \quad x(n)|_{n=0} = x_0, \quad (1)$$

the solution of which is $x(n) = a^n x_0 + b \sum_{j=0}^{n-1} a^j \xi(n-1-j)$. If $x(n)|_{n=n_0 \geq 1} = x_0$, then $x(n) = a^{n-n_0} x_0 + b \sum_{k=n_0}^{n-1} a^{n-(k-1)} \xi(k)$; if $n_0 \rightarrow -\infty$ and $|a| < 1$, then $x(n) = b \sum_{k=-\infty}^{n-1} a^{n-(k-1)} \xi(k) = b \sum_{p=0}^{\infty} a^p \xi(n-1-p)$ (steady mode),

where $\xi(n)$ is a random sequence with a given expected value $M\xi(n, \omega)$ and known correlation function $K_{\xi\xi}(n, m)$.

Besides, $Mx(n_0, \omega) = Mx_0(\omega)$, $x_0(\omega)$ and $\xi(n, \omega)$ – are independent random variables.

Note that (1) defines the simplest kind of regression signal that is widely used in applications. Most often, when analyzing a system of form (1), $\xi(n)$ is assumed to be a stationary random sequence, moreover $K_{\xi\xi}(n, m) = c\delta_{nm}$, where δ_{nm} is a Kronecker symbol.

From (1), for the steady mode correlation function, we obtain

$$K_{xx}(n, m) = |b|^2 \sum_{p, q=0}^{\infty} a^p \overline{a^q} K_{\xi\xi}(n-1-p, m-1-q), \quad (|a| < 1). \quad (2)$$

Consider some particular cases of (2).

Let $\xi(n)$ be a stationary in a broad sense random sequence. Then

$$K_{xx}(n, m) = |b|^2 \sum_{p, q=0}^{\infty} a^p \overline{a^q} K_{\xi\xi}(n-m-(p-q)) = K_{xx}(n-m),$$

that is $x(n)$ is a stationary random sequence.

1) This fact is well known in applications [4], and the rank of non-stationarity of $x(n)$ is zero.

2) Let $\xi(n)$ be a non-stationary dissipative random sequence of finite non-stationarity rank r , which has an operator image in the corresponding Hilbert space H of the form $\xi(n) = A^n \xi_0$, where A is a limited dissipative operator with a finite non-Hermitian subspace $\dim \overline{\text{Im } AH} = r < \infty$. Then

$$K_{\xi\xi}(n, m) = \sum_{\alpha=1}^r \sum_{\tau=0}^{\infty} \varphi_{\alpha}(n+\tau) \overline{\varphi_{\alpha}(m+\tau)} + K_{\xi\xi}^{(0)}(n-m),$$

where $\varphi(n) = \langle A^n \xi_0, g \rangle$, g is a channel element of the operator A , and for $K_{xx}(n, m)$ we get

$$K_{xx}(n, m) = \sum_{\alpha=1}^r \sum_{\tau=0}^{\infty} \Phi_{\alpha}(n+\tau) \overline{\Phi_{\alpha}(m+\tau)} + K_{xx}^{(0)}(n-m),$$

where $\Phi_{\alpha}(n+\tau) = \sum_{p=0}^{\infty} a^p \varphi_{\alpha}(n-1-p+\tau)$. It follows that $x(n)$ is a non-stationary random sequence which rank does not exceed r .

If we consider unsteady solutions of stochastic difference equation (1), then for $K_{xx}(n, m)$ we have

$$K_{xx}(n, m) = |b|^2 \sum_{j, l=0}^{n-1, m-1} a^j \overline{a^l} K_{\xi\xi}(n-1-j, m-1-l). \quad (3)$$

It can be seen from (3) that even when $\xi(n)$ is a stationary random sequence, $x(n)$ is a non-stationary sequence.

Consider the case when $K_{\xi\xi}(p, q)$ is of the form:

$$K_{\xi\xi}(p, q) = \sum_{\alpha=1}^r \psi_{\alpha}(p) \overline{\psi_{\alpha}(q)}, \tag{4}$$

where $\psi_{\alpha}(p) = \langle A^p \xi_0, e_{\alpha} \rangle$, e_{α} is a basis in the Hilbert space H .

Equality (4) is a finite-dimensional analogue of the Karhunen – Loeve decomposition for random sequences [10]. Then from (3) given (4) we obtain:

$$K_{xx}(n, m) = |b|^2 \sum_{\alpha=1}^r \Psi_{\alpha}(n) \overline{\Psi_{\alpha}(m)}, \tag{5}$$

where $\Psi_{\alpha}(n) = \sum_{j=0}^{n-1} a^j \psi_{\alpha}(n-1-j)$.

The rank of non-stationarity in this case does not exceed $2r$, since each summand in (4) or (5) generates two summands in the correlation difference. For example:

if $K_{\xi\xi}(p, q) = \psi(p) \overline{\psi(q)}$, then $K_{xx}(p, q) = \Psi(n) \overline{\Psi(m)}$, $\Psi(n) = \sum_{j=0}^{n-1} a^j \psi(n-1-j)$ and

$$W(n, m) = K(n, m) - K(n+1, m+1) = \Psi(n) \overline{\Psi(m)} - \Psi(n+1) \overline{\Psi(m+1)} = \sum_{\alpha, \beta}^2 \Theta_{\alpha}(n) J_{\alpha\beta} \overline{\Theta_{\beta}(m)},$$

where $\Theta_1(n) = \Psi(n)$, $\Theta_2(n) = \Psi(n+1)$, that is $\Theta_2(n+1) = \Theta_1(n)$, and the matrix J has the form $J = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (in the general case $\Psi(n)$ and $\Psi(n+1)$ are linearly independent).

It can be seen that the rank of non-stationarity does not exceed two. It is equal to one in the case when $\Psi(n)$ and $\Psi(n+1)$ are linearly dependent functions.

Consider a Hankel random sequence $\xi(n)$ with $K_{\xi\xi}(n, m) = K_{\xi\xi}(n+m)$. Then in the steady mode we obtain from (2) that $K_{xx}(n, m) = |b|^2 \sum_{p, q=0}^{\infty} a^p \overline{a^q} K_{\xi\xi}(n+m-p-q-2)$, hence $K_{xx}(n, m)$ depends only on $n+m$. If $\xi(n)$ is of

finite quasirank, that is $K_{\xi\xi}(n, m)$ is represented in the form: $K_{\xi\xi}(n, m) = \sum_{\alpha=1}^{\rho} \sum_{l=0}^{\infty} \varphi_{\alpha}(n-l) \overline{\varphi_{\alpha}(m+l-1)} + K_{\xi\xi}^{(0)}(n+m)$,

then we obtain a similar image for $K_{xx}(n, m)$:

$$K_{xx}(n, m) = \sum_{\alpha=1}^{\rho} \sum_{l=0}^{\infty} \Theta_{\alpha}(n-l) \overline{\Theta_{\alpha}(m+l-1)} + K_{xx}^{(0)}(n+m),$$

where $\Theta_{\alpha}(n) = \sum_{p=0}^{\infty} a^p \varphi_{\alpha}(n-p-1)$.

Remark. In the unsteady mode $K_{xx}(n, m) = \sum_{j, l=0}^{n-1, m-1} \sum_{\tau=0}^{\infty} \Phi(n, \tau) \overline{\Phi(m, \tau)}$, where $\Phi(n, \tau) = b \sum_{j=0}^{n-1} a^j \varphi_{\alpha}(n-1-j+\tau)$.

Since $W(n, m) = |b|^2 \sum_{\tau=0}^{\infty} \Phi(n, \tau) \overline{\Phi(m, \tau)} - |b|^2 \sum_{\tau=0}^{\infty} \Phi(n+1, \tau) \overline{\Phi(m+1, \tau)}$, then the rank of the corresponding quadratic form in the general case is infinite.

Consider a linear system in the case of continuous time t , which is described by a stochastic differential equation of the n -th order of the form:

$$L_t \xi(t) = \eta(t), \tag{6}$$

where $L_t = \sum_{k=0}^n a_k \frac{d^k}{dt^k}$, and $\eta(t)$ is a random process with a known mathematical expectation (let it be zero for simplicity)

and a correlation function $K_{\eta\eta}(t, s)$. Taking into account the corresponding initial conditions at $t = t_0$, the solution of equation (6) has the form:

$$\xi(t) = \xi_{\text{uniform}}(t) + \int_0^t G(t-s)\eta(s)ds, \tag{7}$$

where $G(t-s)$ is the corresponding Cauchy (Green's) function. If all the roots of the characteristic equation $\sum_{k=0}^n a_k \lambda^k = 0$ are negative or have a negative real part, then in (7) one can pass to the limit for $t_0 \rightarrow \infty$ and obtain the steady mode:

$$\xi_{\text{steady}}(t) = \int_{-\infty}^t G(t-s)\eta(s)ds. \tag{8}$$

It follows from (8) that if $\eta(s)$ is a stationary random process, then $\xi_{\text{steady}}(t)$ is also a stationary random process. This simple reasoning is the basis for solving many applied problems of the correlation theory of stationary random processes (linear problems of filtering theory, forecasting, and others). [2 – 4, 7, 11 – 15].

We note that if $\eta(s)$ is a non-stationary dissipative random process of finite rank of non-stationarity, then it follows from (8) that $\xi_{\text{steady}}(t)$ is a non-stationary random process of finite rank of non-stationarity that does not exceed

the rank of the non-stationarity of $\eta(s)$. Indeed, since $K_{\eta\eta}(t, s) = \sum_{\alpha=1}^r \int_0^\infty \varphi_\alpha(t+\tau)\overline{\varphi_\alpha(s+\tau)}d\tau$ then we obtain for

$K_{\xi\xi}(t, s)$ the images $K_{\xi\xi}(t, s) = \sum_{\alpha=1}^r \int_0^\infty \Phi_\alpha(t+\tau)\overline{\Phi_\alpha(s+\tau)}d\tau$, where $\Phi_\alpha(t+\tau) = \int_0^\infty G(\sigma_1)\varphi_\alpha(t+\tau-\sigma_1)d\sigma_1$. In this

case, the infinitesimal correlation function has the form $V_{\xi\xi}(t, s) = \sum_{\alpha=1}^r \Phi_\alpha(t)\overline{\Phi_\alpha(s)}$. The statement is also true for non-

dissipative non-stationary random processes, where $K_{\eta\eta}(t, s)$ is of the form $K_{\eta\eta}(t, s) =$

$\int_0^\infty \sum_{\alpha, \beta=1}^r \varphi_\alpha(t+\tau)J_{\alpha\beta}\overline{\varphi_\beta(s+\tau)}d\tau$, with the involutive matrix $J = (J_{\alpha\beta})$, such that $J^2 = I, J^* = J$.

If in equation (6) the coefficients a_k depend on t , then the rank of the non-stationarity is, in general, not finite, be-

cause $\xi(t)$ takes the form (under zero initial conditions) $\xi(t) = \int_{t_0}^t G(t-s)\eta(s)ds$ and

$$K_{\xi\xi}(t, s) = \int_{t_0}^t \int_{t_0}^t G(t, \sigma_1)\overline{G(s, \sigma_2)}K_{\eta\eta}(\sigma_1, \sigma_2)d\sigma_1d\sigma_2 = \sum_{\alpha=1}^r \int_0^\infty \Phi_\alpha(t, \tau)\overline{\Phi_\alpha(s, \tau)}d\tau,$$

where $\Phi_\alpha(t, \tau) = \int_{t_0}^t G(t, \sigma_1)\varphi_\alpha(\sigma_1+\tau)d\sigma_1$ and the rank of the quadratic form $\sum_{\alpha, \beta=1}^r V_{\xi\xi}(t_\alpha, t_\beta)a_\alpha\overline{a_\beta}$, generally

speaking, is infinite, since $V_{\xi\xi}(t, s) = -\sum_{\alpha=1}^r \int_0^\infty \left(\frac{\partial \Phi_\alpha(t, \tau)}{\partial t} \overline{\Phi_\alpha(s, \tau)} + \Phi_\alpha(t, \tau) \frac{\partial \Phi_\alpha(s, \tau)}{\partial s} \right) d\tau$.

If we turn to linear transformations of fields, a sufficiently general linear transformation of the field $\xi(x)$ ($x = (x_1, \dots, x_n)$) has the image:

$$\xi(x_1, \dots, x_n) = \int_V G(x, y)\eta(y)dy, \tag{9}$$

where $G(x, y)$ is Green's function of the corresponding differential or integro-differential operator, and $\eta(y)$ is the field of given random sources. If $G(x, y) = G(x-y)$, and $V = R_n$, then

$$\xi(x) = \int_{R_n} G(x-y)\eta(y)dy. \tag{10}$$

It follows from (9) that $\xi(x)$ is a statistically homogeneous field if $\eta(y)$ is a statistically homogeneous field. If the field $\eta(y)$ has a finite rank of inhomogeneity r , then, as in the case of non-stationary random processes, it is easy to show that the field $\xi(x)$ has a rank of inhomogeneity that does not exceed r .

For continuous Hankel random processes and fields, it is also true that the solutions of the form (7), (10), which

can be considered as linear transformations of the Hankel process or Hankel field, preserve *the property of Hankel invariance*, and random processes and fields of finite quasirang are mapped into random processes and fields of finite quasirang.

The following statement is crucial for further study. If we consider the general linear problem for ξ of the form:

$$L\xi(x) = \eta(x), \quad (11)$$

where L is an arbitrary linear deterministic operator, and $\eta(x)$ is a random process or a field with a continuous correlation function, then embedding $\eta(x)$ into corresponding Hilbert space H_η we also obtain a corresponding embedding of $\xi(x)$ in H_η , and stochastic problem (11) turns into a linear deterministic problem in H_η of the form $L\xi_x = \eta_x$. Within the framework of the correlation analysis of this problem, the corresponding correlation function is represented by a scalar product:

$$K_{\eta\eta}(x, y) = \langle \eta_x, \eta_y \rangle_{H_\eta}, \text{ and } K_{\xi\xi}(x, y) = \left\langle \left(L^{-1}\xi \right)_x, \left(L^{-1}\xi \right)_y \right\rangle_{H_\eta}.$$

This transition follows directly from Karhunen – Loeve orthogonal decomposition and can be used for solving linear stochastic problems.

Prospects for further research. The results obtained can easily be transferred to the case of vector equations of the form (1), where a and b are matrices of the corresponding order, or when a and b are operators in auxiliary variables. The proposed approach can be used to model non-stationary random sequences and their classification in the framework of correlation theory.

Conclusions. Thus, the correlation differences introduced in [8, 9] allow one to study quite completely the non-stationarity structure of some classes of random sequences, which are determined by stochastic difference equations. The Hilbert approach to the correlation theory of random sequences allowed us to study the question of the asymptotic behavior of the correlation function and made it possible to obtain a simple inhomogeneous representation of the correlation function in terms of the correlation difference.

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СИСТЕМНИЙ МОДУЛЬ «РЕЗИСТЕНТНІСТЬ» НЕЧІТКОЇ ЕКСПЕРТНОЇ СИСТЕМИ ПРОФЕСІЙНИХ РИЗИКІВ

Системний модуль «Резистентність» експертної системи з нечітким логічним виведенням побудовано за результатами якісного та кількісного аналізу професійних ризиків стану здоров'я робочих підприємств хіміко-фармацевтичної галузі Харкова та Харківської області. Модуль «Резистентність» є складовою частиною нечіткої експертної системи «РИЗИК» оцінки ризику розвитку професійно обумовлених захворювань. Якісний аналіз ризиків надав можливість ідентифікувати найбільш впливові на стан здоров'я фактори ризику. Запропоновано метод кількісної оцінки професійних ризиків в умовах неповноти інформації на основі аналізу кривих оцінки резистентності гомеостазу. Метод може бути використаний для диференціації рівнів ризику розвитку широкого кола захворювань.

Ключові слова: нечітка експертна система, якісний та кількісний аналіз ризику, професійні ризики, криві резистентності гомеостазу.

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СИСТЕМНЫЙ МОДУЛЬ «РЕЗИСТЕНТНОСТЬ» НЕЧЕТКОЙ ЭКСПЕРТНОЙ СИСТЕМЫ ПРОФЕССИОНАЛЬНЫХ РИСКОВ

Системный модуль «Резистентность» экспертной системы с нечетким логическим выводом построен по результатам качественного и количественного анализа профессиональных рисков состояния здоровья рабочих предприятий химико-фармацевтической отрасли Харькова и Харьковской области. Модуль «Резистентность» является составной частью нечеткой экспертной системы «РИСК» оценки риска развития профессионально обусловленных заболеваний. Качественный анализ рисков дал возможность идентифицировать факторы риска, оказывающие наибольшее влияние на состоянии здоровья. Предложен метод количественного анализа профессиональных рисков в условиях неполноты информации на основе кривых оценки резистентности гомеостаза. Метод может быть использован для дифференциации уровней риска развития широкого круга заболеваний.

Ключевые слова: нечеткая экспертная система, качественный и количественный анализ риска, профессиональные риски, кривые резистентности гомеостаза.

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«RESISTANCE» SYSTEM MODULE OF PROFESSIONAL RISK FUZZY EXPERT SYSTEM

The «Resistance» system module of an expert system with a fuzzy logical output is based on the results of the qualitative and quantitative analysis of the occupational health risks of the chemical and pharmaceutical industries workers in Kharkov and Kharkov region. The «Resistance» module is an integral part of the «RISK» fuzzy expert system for assessing the risk of developing professionally caused diseases. Qualitative risk analysis made it possible to identify risk factors that have the greatest impact on the health status. A method is proposed for a quantitative analysis of occupational risks under conditions of incomplete information on the basis of curves for assessing the resistance of homeostasis. The method can be used to differentiate risk levels for the development of a wide range of diseases.

Key words: fuzzy expert system, qualitative and quantitative risk analysis, professional risks, homeostasis resistance curves.

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